# YPP December 2012: Angular Momentum Makes the World Go Round

## Laboratory

### Introduction

The purpose of this lab is to study the various aspects of rotation to determine how shape, size, mass, or distribution of mass affects the motion of objects rolling down an inclined plane. Take some time to discuss: What is angular momentum? Does it have direction? What is a moment of inertia? Is angular momentum conserved? What does this mean? If it is conserved, how do we get things spinning in the first place?

### Angular Acceleration of a Rotating Disk

This experiment uses a disk rotating about a vertical axis through the center of the disk. A string is tied to a pulley which is attached to the disk. The disk experiences a torque, $\tau$, provided by a hanging mass which is attached to the other end of the string as shown in the figure to the right. You will be examining the angular acceleration of two shapes: a solid disk and a thick hollow ring. The hanging mass would fall at a rate of 9.8 $m/s^2$ except that it has a tension force due to the string that is slowing it down. Our goal is to find the acceleration value of the hanging mass, $a_y$, by measuring the time it takes for the mass to fall a given value, $\Delta y$. The distance that the hanging mass (which is initially at rest) will fall is given by the following equation:

$$\Delta y = \frac{1}{2}at^2 \tag{1}$$

There are two forces causing this acceleration: the force due to gravity on the hanging mass and the tension force due to the string. The tension force is also creating torque on the rotating disk which is causing an angular acceleration. We can calculate the torque due to the tension force from the following equation:

$$\sum \tau = \mathbf{r} \times \mathbf{F} \tag{2}$$

Where r is the radius value is the distance from the central axis of the rotating disk to the place where the string attaches. What this means is that you will get more torque by both increasing how hard you push on the object, but also by pushing farther away from the axis of rotation. You have certainly noticed that it is much easier to open a door by pushing far from the hinge as opposed to close to it.

In addition to where you push on a door, the shape and mass distribution of the door are also very important for determining how easy it will be to open. The measure of how easy or hard something is to turn is called its moment of inertia, and is a function of the mass of an object and its distribution. To get an idea of why this is, condsider a point of mass near the inside of a spinning disk and another mass near the outside. In order for the disc to spin without deforming or breaking, the small bit of mass near the outside of the disc needs to move faster than the piece of mass near the inside. The speed, v, a mass is going at some rate of rotation $\omega$ relates linearly with the distance from the axis of rotation.

$$v = \omega \times r \tag{3}$$

Similarly the angular acceleration is related to the linear acceleration of any point in the object.

$$a = \alpha \times r \tag{4}$$

Since we know from last month the amount of energy of a moving object has is related to the square of its velocity, each little piece of mass has the following energy associated with it.

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2 \tag{5}$$

If we add all these pieces of mass together we find that the total energy something has while spinning is:

$$E_{rot} = \frac{1}{2}\omega^2 \sum mr^2 = \frac{1}{2}I\omega^2 \tag{6}$$

This quantity I, the moment of inertia, is the measure of how the distribution of mass changes how hard something is to rotate. Just like how a massive object is harder to accelerate then a lighter object (you need more force), an object with a larger moment of inertia will be harder to get spinning then something with a smaller moment of inertia.

In the case of linear acceleration we have Newton's laws to go off of:

$$\sum F = ma \tag{7}$$

In the case of angular acceleration we can simply replace each element by its equivalent version, namely: force $\rightarrow$ torque, mass $\rightarrow$ moment of inertia, and acceleration $\rightarrow$ angular acceleration.

$$\sum \tau = I\alpha \tag{8}$$

In this lab we will be determining the moments of inertia of a couple of objects. Using some math, one could calculate what some of these values should be based on knowledge about how the mass of the object is distributed. The moment of intertia of a thick hollow ring is:

$$I_{ring} = \frac{1}{2}m(R_1^2 + R_2^2) \tag{9}$$

Where $R_1$ is the radius of the inside of the ring and $R_2$ is the radius of the outside of the ring. The moment of inertia of a solid disk is:

$$I_{disk} = \frac{1}{2}mR^2 \tag{10}$$

However you shouldn't trust me that these are correct, so lets do an experiment to calculate them. Further, suppose you take a wierdly shaped pulley and wanted to calculate its moment of inertia, it would be difficult to do by summing up all the little bits of mass.

1. Start by measuring the distance that the mass will drop when you are conducting the experiment. The mass will start at some initial value and drop to a final value due to the length of the string. This measured value will be your $\Delta y$ distance for your equation.

2. Make sure the string passes over the pulley and attaches to the solid disk. Now perform the experiment by taking the mass up to its initial value and releasing it. Measure the time it takes for the mass to fall the distance $\Delta y$ with a stopwatch. Repeat the experiment five times is order to get an accurate time value. Record all five time values. Take the average value for time to calculate the acceleration of the hanging mass.

3. From the value of acceleration found in step 2, calculate the tension force due to the string on the hanging mass. (Hint: you can easily calculate the value for the force due to gravity on the hanging mass.)

4. Measure the distance from the central axis to the location of the string that is wrapped around the rotating disk. This is the radius value that is providing the torque that is making the disk rotate. Calculate the magnitude of torque on the disk from the tension force and radius value you found.

5. Using the relationship between linear acceleration and angular acceleration, calculate the angular acceleration of the disk or ring.

6. Calculate the moment of inertia by dividing the total torque found in step 4 by the angular acceleration found in step 5.

7. Calculate the moment of inertia of each object using equations 8 and 9. Did you find values close to what your exeriment found? Why might they be a bit different?

**Shape Races**

In this experiment, you will take a variety of objects and try to race them down a straight slope. For each race make sure you release the objects at the same height and at the same time (a ruler usually accomplishes this task well enough). There are six objects that you will be racing: a solid sphere, a hollow ring, a large radius solid aluminum disk, a small radius solid aluminum disk, a small radius solid copper disk, and a small radius solid plastic disk. Make sure you have all six objects. Race the objects in pairs.

Discuss as a group which shapes would win for the following object races:

1. A large radius solid aluminum disk vs. a hollow ring.
2. A large radius solid aluminum disk vs. a solid shere.
3. A solid sphere vs. a hollow ring.
4. A small radius solid aluminum disk vs. a hollow ring.
5. A small radius solid aluminum disk vs. a large radius solid aluminum disk.
6. A large radius solid aluminum disk vs. a small radius plastic disk.
7. A small radius solid copper disk vs. a small radius solid plastic disk.

Which shape in your collection is the overall fastest? Which shape in your collection is the overall slowest?

Which shape in your collection has the greatest moment of inertia? Which shape in your collection has the least moment of inertia?

## Demos

### Bicycle Wheel

- Spin the wheel up with a dremel tool or by hand.
- Have the students find how difficult it is to move the wheel while it is spinning compared to when it is not.
- While sitting in a chair, have a student flip the wheel upside down, causing them to start to spin.

### Weights and Chair

- Give a quick explanation of moment of inertia.
- Have a student sit in a chair and have another student gently spin them up while they are spinning have them move the weights in and out and watch how their rotation rate changes.

### Tops

- Draw free body diagram, show the torques and angular momentum vectors and the new angular momentum vector after a short time. Actually spin tops, then poke them to show precession due to a weak torque. If you need help explaining why the top precesses, just ask, or there may be a mechanics book hanging around (just think of the direction of the Tops force applied, then the direction of the torque, dL/dt = torque, and thus which way the top must precess to keep this true with a gravitational torque).

- Talk about the torque on the earth that causes it to precess (it comes from the earth not being a perfect sphere, and the sun and moon pulling on the off (solar) axis bulges).

**Gyroscopes**

- Gyroscopes relay on conservation of angular momentum as well. Most gyroscopes are mounted in a gimbal set so that if the gimbal set is locked onto, say, a ship, the gyroscope, which can be as simple as a spinning disc, does not move when the ship pitches or yaws. The gimbals move, but the gyroscope stays relatively stationary because of its angular momentum. Mounting three gyroscopes spinning in different directions can allow very sensitive equipiment to maintain the correct direction in ships and planes.

- Spin up the gyroscopes available, either by hand or with a dremel tool. Show that it is difficult to turn the gyroscopes when a torque is applied- hence their use. Talk about other applications (IMUs, ship navigation, rocket engines, compasses, Hubble, ICBMs, Wiimotes, etc.)

**Movies!**

http://www.youtube.com/watch?v=gdAmEEAiJWo